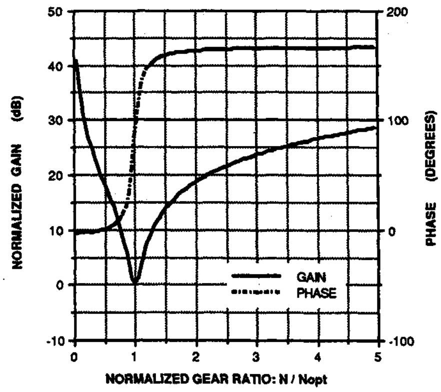

Fig. 2 Normalized magnitude and phase of Eq. (9) as a function of the normalized gear ratio.

Table 1 System parameters

$J_c = 1000.0$, kg-m^2	$D_c = 1.0$, N-m-s/rad
$J_0 = 5.0$, kg-m^2	$D_0 = 0.6$, N-m-s/rad
$J_1 = 20.0$, kg-m^2	$D_1 = 1.2$, N-m-s/rad
$K = 10.0$, N-m/rad	$C = 1.0$, W/N-m^2

(10)] will simultaneously minimize both the dynamic coupling between the spacecraft and the appendage [Eq. (9)] and the overall actuator power consumption [Eq. (11)].

Determination of the Optimum Coupling Ratio

The magnitudes of Eqs. (9) and (10) were expressed in the frequency domain ($s = j\omega$) and then differentiated with respect to N. Solution of these equations set equal to zero, neglecting viscous damping, resulted in the following condition for N at all frequencies:

$$N_{\text{opt}} = \sqrt{\frac{J_1}{J_0}} \tag{12}$$

Numerical Example

Table 1 gives representative parameter values for the system shown in Fig. 1, which was analyzed to show the result shown in Eq. (12) for a system with arbitrary viscous damping. Using these parameters, the maximum magnitude of Eqs. (9) and (10) as a function of N were calculated choosing driving frequencies five decades above and below the system natural frequency for incremental values of N. For every frequency, a distinct minimum occurred for $N = N_{\text{opt}}$ as given by Eq. (12). To show this effect, the gain (normalized to its magnitude at $N = N_{\text{opt}}$ and phase of Eq. (10) for a given frequency $s = j\omega$ are plotted as a function of the normalized coupling ratio in Fig. 2.

Results and Conclusion

Figure 2 shows the dramatic improvement in motor torque to appendage transmission for any given motion achieved by selecting the optimum gear coupling ratio as shown in Eq. (12). Using this method, typical lightly damped spacecraft appendages will exhibit improved controllability, a reduced level of required power during a maneuver, reduced dissipation, and reduced interaction with the attitude control dynamics.

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Reduction of Missile Navigation Errors by Roll Programming

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Introduction

THE concept of rotating inertial instruments within a vehicle to allow averaging or cancellation of their errors has often been applied in the past. Stable platforms of the "carousel" type include an auxiliary turntable rotating continuously around the yaw axis, on which are mounted the instruments associated with the roll and pitch axes. (The yaw gyro is not mounted on the turntable, because its scale factor errors would eventually result in large yaw attitude errors.) And at least one strapdown system[1] rotates all of its instruments back and forth through ±360 deg. In both cases the rate of rotation is typically about 1 rpm.

In a missile employing strapdown instruments for navigation, a natural way of using the same principle is by changing the roll attitude of the missile itself, so that no hardware alterations are necessary. In this case, continuous rotation in one direction is not desirable because of the roll gyro scale factor (SF) error. Back and forth rotation will largely eliminate this effect, provided the SF error is a symmetrical one; for unsymmetrical SF errors it is desirable to minimize the total amount of roll motion. In addition, uplink communications may require upright (or inverted) roll attitudes for reasons of antenna polarization. The strategy considered here consists of simply introducing one or more periods of inverted flight into the trajectory, attained by relatively rapid rollovers so that the missile is nearly always either upright or inverted. In general, most of the achievable improvement can be realized by a simple schedule (such as a single inverted interval), preprogrammed at launch, based on the expected time of flight.

The technique can be applied to the reduction of chosen components of navigation error or, in the case of a terminal homing missile, to reduce seeker pointing errors (SPE) or heading errors at the expected time of target acquisition. We will concentrate here on reducing SPE. Further details can be found in Ref. 2.

The choice of an appropriate roll schedule is facilitated by deriving "influence functions" $h(t)$ that indicate the effect, on each component of SPE, of each reversible instrument error coefficient c_i:

$$\text{SPE}_i = \int_0^T c_i(t) h_i(t)\, dt \tag{1}$$

In Eq. (1), the coefficient c_i is constant in magnitude, but its sign changes when the missile is inverted. If we introduce the sign function

$$u(t) = \begin{cases} +1 & \text{(missile upright)} \\ -1 & \text{(missile inverted)} \end{cases} \tag{2}$$

we may express Eq. (1) as

$$\text{SPE}_i = c_i \int_0^T u(t) h_i(t)\, dt \tag{3}$$

where c_i is now constant.

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Knowing the influence function $h(t)$ for each important error source facilitates the choice of a roll program [defined by $u(t)$] that will simultaneously drive to zero (or nearly to zero) each integral of the given type. Although there may be a considerable number of reversible error coefficients, usually a few error sources predominate so that only a small number of influence functions must be dealt with.

Susceptibility of Instrument Errors to Roll Reversal

Not all of the various types of inertial instrument errors are susceptible to cancellation by this technique, but those that are include some that are often very important, such as accelerometer biases and constant gyro drifts. Whether or not a particular error component is reversed in sign when the missile is inverted depends on 1) whether the input axis (IA) of the instrument is reversed in direction, and 2) whether the sign of the error, relative to the IA, is reversed (e.g., because of a reversal of the acceleration vector relative to the instrument axes). We can thus develop a set of rules for determining which errors are reversed when the missile is inverted in roll:

1) Bias drifts and accelerometer biases on the yaw and pitch axes are reversed, but those on the roll axis are not.

2) Gyro and accelerometer scale factor errors are not reversed.

3) A g-sensitive drift is reversed if it involves the roll axis and either the yaw or pitch axis. If we identify such a drift by (i,j) where i is the axis of drift and j the axis of the exciting specific force, and let roll = 1, yaw = 2, and pitch = 3, then the reversible g-sensitive drifts are those designated 12, 13, 21, 31, while those not reversed are 11, 22, 33, 23, 32.

4) A g^2-sensitive drift can be designated by (i,j,k), where i is the axis of drift and j and k are the axes of the specific forces whose product $f_j f_k$ causes the drift. The drift will be reversed if either 1) one acceleration axis is reversed ($11k$; $k \neq 1$), 2) the IA is reversed ($i11$; $i \neq 1$), or 3) the IA and *both* acceleration axes are reversed (ikj; $i,j,k \neq 1$). Thus, the reversible g^2-sensitive drifts are those designated 112, 113; 211, 311; 222, 223, 322, 233, 323, 333. Those not reversed are 111, 122, 133, 123; 212, 213, 312, 313.

5) Errors due to misalignments of gyros or accelerometers around the yaw or pitch axes are reversed, but roll misalignments are not.

Derivation of Influence Functions

For simplicity, we consider only planar missile trajectories, as shown in Fig. 1. We define two right-handed Cartesian coordinate frames, the M and L frames. The M frame is the missile body frame with axes along $M1$ = roll, $M2$ = yaw, $M3$ = pitch. The L frame is an inertial frame oriented relative to the line of sight (LOS) to the target at the time of acquisition, as shown. The $L1$ axis is toward the target, $L2$ upward, and $L3$ horizontal.

The linear differential equations that describe the propagation of the errors in missile position, velocity, and attitude may be written approximately in the form[3]

$$\mathrm{d}/\mathrm{d}t \begin{bmatrix} \delta x^{(L)} \\ \delta \dot{x}^{(L)} \\ \delta \phi^{(L)} \end{bmatrix} = \begin{bmatrix} 0 & I & 0 \\ 0 & 0 & -[f^{(L)}\times] \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \delta x^{(L)} \\ \delta \dot{x}^{(L)} \\ \delta \phi^{(L)} \end{bmatrix} + \begin{bmatrix} 0 \\ C\delta f^{(M)} \\ C\delta\dot{\phi}^{(M)} \end{bmatrix} \tag{4a}$$

or

$$\dot{z} = Fz + Bc \tag{4b}$$

where c is the vector whose elements are the instrument error coefficients and $f^{(L)}$ is the vector of specific forces acting on the missile, expressed in the L frame. The symbol $[f^{(L)}\times]$ represents a "cross-product matrix" such that its product with any vector v is the vector $f^{(L)} \times v$. The matrix C is the transformation matrix from the M frame to the L frame,

$$C = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{5}$$

$\delta f^{(M)}$ is the vector of specific force (acceleration) errors, expressed in the M frame, and $\delta\dot{\phi}^{(M)}$ is the vector of drift rates (gyroscope errors). Each element of these two vectors is a linear combination of the instrument error coefficients, each multiplied by some function of missile angular rate components or specific force components.

These equations are based on the approximation of a nonrotating flat Earth and are therefore valid only for flight times that are short compared with the 84-min Schuler period. The simple form of Eqs. (4) makes possible analytic solution of the associated adjoint equations.

Our main interest here is in the yaw and pitch components of SPE, which can be written as linear combinations of the elements of the final state perturbation vector $z(T)$:

$$e_{\text{yaw}} = k_y^T z(T) \tag{6}$$

$$e_{\text{pitch}} = k_p^T z(T) \tag{7}$$

where

$$k_y^T = [0 \quad 0 \quad -1/R \quad 0 \quad 0 \quad 0 \quad 0 \quad 1 \quad 0] \tag{8}$$

$$k_p^T = [0 \quad 1/R \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 1] \tag{9}$$

and R is the range from missile to target at acquisition. The corresponding influence functions can be determined by means of adjoint theory.[4] If the adjoint vector $\lambda(t)$ obeys the differential equation

$$\dot{\lambda} = -F^T\lambda \tag{10}$$

with the terminal boundary condition

$$\lambda(T) = k \tag{11}$$

then it follows that the SPE components of Eqs. (6) and (7) can be computed in the form

$$e = k^T z(T) = \lambda^T(0)z(0) + \int_0^T h^T(t)c(t)\,\mathrm{d}t \tag{12}$$

where

$$h(t) = B^T(t)\lambda(t) \tag{13}$$

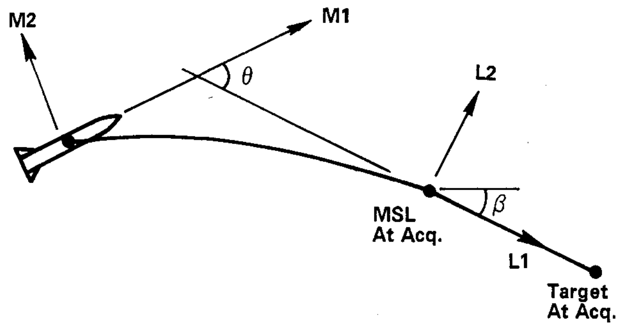

Fig. 1 Coordinate frames. The *L* frame is inertially fixed, while the *M* frame is tied to the missile body.

is the vector of influence functions. The first term in Eq. (12) represents the effects of initial errors in position, velocity, and attitude, and the integral term is a sum of terms of the form of Eq. (1), one for each instrument error coefficient in the vector c.

Equation (10) represents nine coupled first-order differential equations and in general must be solved twice for the boundary conditions k corresponding to the yaw and pitch SPE. In the present case, however, it turns out that only two quantities must actually be determined by integration:

$$g_1(t) = \frac{-1}{R}\int_t^T t_{go}(\tau) f_{L2}(\tau)\,\mathrm{d}\tau \tag{14}$$

$$g_2(t) = 1 + \frac{1}{R}\int_t^T t_{go}(\tau) f_{L1}(\tau)\,\mathrm{d}\tau \tag{15}$$

where

$$t_{go}(\tau) = T - \tau \tag{16}$$

and f_{L1} and f_{L2} are components of the specific force vector in the L frame. In fact, these integrations are also unnecessary, once the trajectory has been determined. Using the fact that the acceleration vector is equal to $f+g$, it may be shown that

$$g_1(t) = \frac{-1}{R}\left[y_L(T) - y_L(t) - \dot{y}_L(t)t_{go} + \frac{g}{2}t_{go}^2\cos\beta\right] \tag{17}$$

and

$$g_2(t) = 1 + \frac{1}{R}\left[x_L(T) - x_L(t) - \dot{x}_L(t)t_{go} - \frac{g}{2}t_{go}^2\sin\beta\right] \tag{18}$$

where β is the depression angle of the line of sight at target acquisition time T (Fig. 1). Thus, the adjoint approach also provides a very efficient way of computing the various components of the seeker pointing error at a particular time.

Table 1 displays the influence functions corresponding to the 16 reversible error sources considered here, expressed in terms of g_1, g_2, and the two additional functions

$$g_3(t) = g_1\cos\theta + g_2\sin\theta \tag{19}$$

$$g_4(t) = -g_1\sin\theta + g_2\cos\theta \tag{20}$$

Table 1 SPE Influence functions

Error type	No.	Error coefficient	SPE component affected	Influence function
Gyro				
Bias drift	1	d_{02}	Y	g_4
	2	d_{03}	P	g_2
Misalignment	3	G_{12}	Y	$-\omega_{M3}g_2$
g-sensitive drift	4	F_{12}	Y	$f_{M2}g_3$
	5	F_{21}	Y	$f_{M1}g_4$
	6	F_{31}	P	$f_{M1}g_2$
g^2-sensitive drift	7	F_{112}	Y	$f_{M1}f_{M2}g_3$
	8	F_{211}	Y	$f_{M1}^2 g_4$
	9	F_{222}	Y	$f_{M2}^2 g_4$
	10	F_{311}	P	$f_{M1}^2 g_2$
	11	F_{322}	P	$f_{M2}^2 g_2$
Accelerometer				
Bias	12	a_{02}	P	$(t_{go}/R)\cos\theta$
	13	a_{03}	Y	$-t_{go}/R$
Misalignment	14	A_{13}	P	$f_{M2}(t_{go}/R)\sin\theta$
	15	A_{23}	P	$-f_{M1}(t_{go}/R)\cos\theta$
	16	A_{32}	Y	$-f_{M1}(t_{go}/R)$

The tabulated error coefficients may be identified by reference to the susceptibility rules presented previously, noting the definition of the 1, 2, 3 axes in rule 3. In the subscript on a misalignment coefficient, the first digit indicates the input axis of the instrument and the second the axis about which it is misaligned.

Once the important reversible error sources have been identified and the corresponding influence function $h(t)$ determined for each, one is better equipped to search for a roll program $u(t)$ [Eq. (2)] that will simultaneously remove most of the error effects when applied as in Eq. (3). In general far fewer than 16 errors need to be dealt with, since there is little benefit to be obtained from reducing errors that are already unimportant relative to the irreducible (irreversible) ones. In a typical case, e.g., perhaps only the bias errors (1, 2, 12, and 13 in Table 1) would be worth dealing with. And since complete nulling of these effects is not worth striving for, one might use the additional approximation $\theta \approx 0$ to reduce the number of applicable influence functions to two, namely $g_2(t)$ and t_{go}/R. Such an approach is used in the numerical example discussed next.

Roll-Induced Errors

The previous discussions have dealt only with error propagation along a planar trajectory with no missile roll. When a roll program is applied to reduce some of the errors, the act of rolling will itself excite errors that would not be present otherwise. Generally, the only significant one of these errors is that due to the roll gyro scale factor error S_{G1}, which causes a buildup of roll attitude error when the roll rate is nonzero.

If the roll SF error is unsymmetrical (different for positive and negative rotations), little can be done about this effect except to minimize the amount of rolling. For a symmetrical SF error, it can be shown that the result is an additional component of yaw SPE given by

$$e_{yaw} = S_{G1}\sum_{i=1}^{n} g_3(t_i)\Delta\phi_i \tag{21}$$

where the summation is over the discrete roll maneuvers applied at times t_i, and $\Delta\phi_i$ is the associated change in roll angle for each maneuver. Normally, $\Delta\phi_i$ has magnitude $\pm\pi$ rads. However, a roll of $\pm 2\pi$ rad can be applied at any time to alter the roll attitude error without changing the effective attitude of the missile; this can be applied to advantage in many situations. For example, a single period of inverted flight usually involves a constant roll attitude error, which is introduced by the initial rollover and disappears when the missile rights itself by rolling in the opposite sense. A 360-deg roll, at some time during the inverted period, can be used to reverse the sign of the roll attitude error while leaving the missile inverted, and can thus be used to cancel the resulting component of SPE without altering other error propagations.

In these discussions, it has been tacitly assumed that the missile's initial attitude errors are not correlated with the in-flight instrument errors. This will not be true if the inertial system is aligned before launch using its own accelerometers (as in self-leveling to the gravity vector on the ground, or transfer alignment from a launch aircraft's master inertial system in flight). In such cases the initial tilts due to accelerometer biases tend to cancel the in-flight errors due to the same biases if the vehicle attitude is roughly constant. Under these circumstances, inverted flight will interfere with this cancellation and may make these particular error effects worse rather than better. Gyro biases, however, can still be neutralized by roll techniques.

Numerical Example

Table 2 lists the error sources included in a simulated test case and indicates by asterisks which ones are reversed in sign during inverted flight. The numerical values shown might be typical of a missile designed for homing interception rather

Table 2 Error parameters (3-σ) for the example

1	Initial msl align. about *dn*-range axis	1.0 deg
2	Initial msl align. about vertical axis	1.0 deg
3	Initial msl align. about cross-*r* axis	1.0 deg
4	Axial (*M*1) accel bias	16.1 ft/s/s
5[a]	Vert (*M*2) accel bias	16.1 ft/s/s
6[a]	Lateral (*M*3) accel bias	16.1 ft/s/s
7	Axial (*M*1) accel scale factor error	0.10
8	Vert (*M*2) accel scale factor error	0.10
9	Lateral (*M*3) accel scale factor error	0.10
10	Align. of *M*3 (lat) accel about *M*1 axis	0.005 rad
11	Align. of *M*2 (vert) accel about *M*1 axis	0.005 rad
12[a]	Align. of *M*3 (lat) accel about *M*2 axis	0.005 rad
13[a]	Align. of *M*1 (long) accel about *M*2 axis	0.005 rad
14[a]	Align. of *M*2 (vert) accel about *M*3 axis	0.005 rad
15[a]	Align. of *M*1 (long) accel about *M*3 axis	0.005 rad
16	Roll gyro (*M*1) bias drift	0.005 rad/s
17[a]	Yaw gyro (*M*2) bias drift	0.005 rad/s
18[a]	Pitch gyro (*M*3) bias drift	0.005 rad/s
19	Roll gyro (*M*1) scale factor error	0.15
20	Yaw gyro (*M*2) scale factor error	0.15
21	Pitch gyro (*M*3) scale factor error	0.15
22	Align. of *M*3 (pitch) gyro about *M*1 axis	0.005 rad
23	Align. of *M*2 (yaw) gyro about *M*1 axis	0.005 rad
24[a]	Align. of *M*3 (pitch) gyro about *M*2 axis	0.005 rad
25[a]	Align. of *M*1 (roll) gyro about *M*2 axis	0.005 rad
26[a]	Align. of *M*2 (yaw) gyro about *M*3 axis	0.005 rad
27[a]	Align. of *M*1 (roll) gyro about *M*3 axis	0.005 rad
28	Gyro *G*-sens drift 11 (roll/*M*1 accel)	3E-5 r/s/(ft/s/s)
29[a]	Gyro *G*-sens drift 12 (roll/*M*2 accel)	3E-5 r/s/(ft/s/s)
30	Gyro *G*-sens drift 22 (yaw/*M*2 accel)	3E-5 r/s/(ft/s/s)
31	Gyro *G*-sens drift 32 (pitch/*M*2 accel)	3E-5 r/s/(ft/s/s)
32	Gyro G^2-sens drift 111	2E-8 r/s/(ft/s/s)^2
33	Gyro G^2-sens drift 122	2E-8 r/s/(ft/s/s)^2
34[a]	Gyro G^2-sens drift 222	2E-8 r/s/(ft/s/s)^2
35[a]	Gyro G^2-sens drift 233	2E-8 r/s/(ft/s/s)^2
36[a]	Gyro G^2-sens drift 322	2E-8 r/s/(ft/s/s)^2
37[a]	Gyro G^2-sens drift 333	2E-8 r/s/(ft/s/s)^2
38	Seeker gimbal yaw error	0.01 rad
39	Seeker gimbal pitch error	0.01 rad

[a]Susceptible to roll reversal.

Table 3 Seeker pointing errors (deg 3-σ), with and without roll programming

		SPE comp	Upright	With roll
2	Initial msl align. about vert axis	Y	2.2	2.2
3	Initial msl align. about cr-r axis	P	2.1	2.1
4	Axial (*M*1) accel bias	P	8.0	8.0
5[a]	Vert (*M*2) accel bias	P	20.1	0.7
6[a]	Lateral (*M*3) accel bias	Y	20.9	0.0
7	Axial (*M*1) accel scale factor error	P	4.1	4.1
8	Vert (*M*2) accel scale factor error	P	1.4	1.4
16	Roll gyro (*M*1) bias drift	Y	5.1	5.1
17[a]	Yaw gyro (*M*2) bias drift	Y	17.2	1.0
18[a]	Pitch gyro (*M*3) bias drift	P	18.0	0.9
21	Pitch gyro (*M*3) scale factor error	P	5.3	5.3
28	Gyro *G*-sens drift 11 (roll/*M*1 accel)	Y	1.3	1.3
	Total *rss* SPE:	Y	27.7	5.9
		P	29.0	10.6

[a]Susceptible to roll reversal.

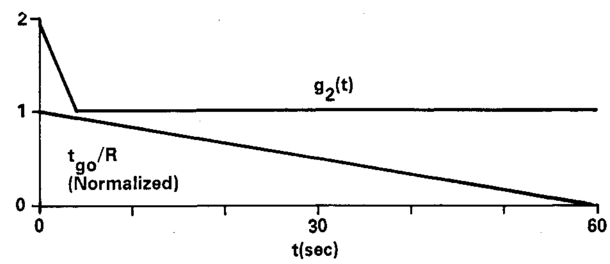

Fig. 2 SPE influence functions for accelerometer biases (t_{go}/R) and gyro biases [$g_2(t)$] for $\theta = 0$.

than inertial navigation. Positions and velocities of target and launcher were assumed to be accurately known so that the errors included are only those due to inertial instrument errors, initial alignment errors, and seeker gimbal errors. (In a real scenario, targeting errors can be quite significant; their effects can be added to the errors given here in RSS fashion.) The effects of roll gyro scale factor errors in such a maneuver are not included, but techniques for eliminating these effects have been discussed.

The missile is air-launched at 500 fps at a flight-path angle $\gamma = 25$ deg. Boost occurs at zero pitch rate and 12 *g* of axial acceleration and lasts for 4 s to yield a ΔV of 1544 fps. The remainder of the 60-s flight to acquisition is at constant velocity (zero axial acceleration) and a constant pitch rate that pitches the trajectory downward by 35 deg before target acquisition at $\gamma = -10$ deg. Intercept takes place after 25 s of additional straight-line, constant-velocity flight. The target is incoming at 1000 fps, and the acquisition range R is 75,853 ft.

For this case, it turns out that the major sources of error are the gyro and accelerometer biases on the yaw and pitch axes. If the angle θ were identically zero, the normalized influence functions of concern would therefore be $g_2(t)$ and t_{go}/R. If we also neglect the axial specific force $f_{L1}(t)$, except during boost, the two influence functions appear as sketched in Fig. 2. Except for the small nonconstant portion of $g_2(t)$ during the boost phase, the integrals of both of these functions can be zeroed by reversing signs in the middle half of the trajectory, from 15 to 45 s, by flying the missile in an inverted position during this interval. This simple strategy was applied directly to the intercept previously described, even though θ is as large as about 35 deg at times.

Table 3 shows all SPE components of significance (> 1 deg 3-σ) when the missile flies upright all the way, together with the corresponding values when the simple roll strategy is used. The four SPE components caused by reversible error sources (numbers 5, 6, 17, and 18) are all reduced by at least a factor of 17. When combined in RSS fashion with all the other components (including those too small to be included in Table 3), the result is an SPE reduction by a factor of about five in yaw and three in pitch.

Conclusions

Simple missile roll strategies have been shown to provide a powerful means of reducing midcourse navigation errors and seeker pointing errors at target acquisition. This allows increased range of operation for a given quality of inertial instruments or, conversely, the use of less expensive instruments for a given level of required performance.

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